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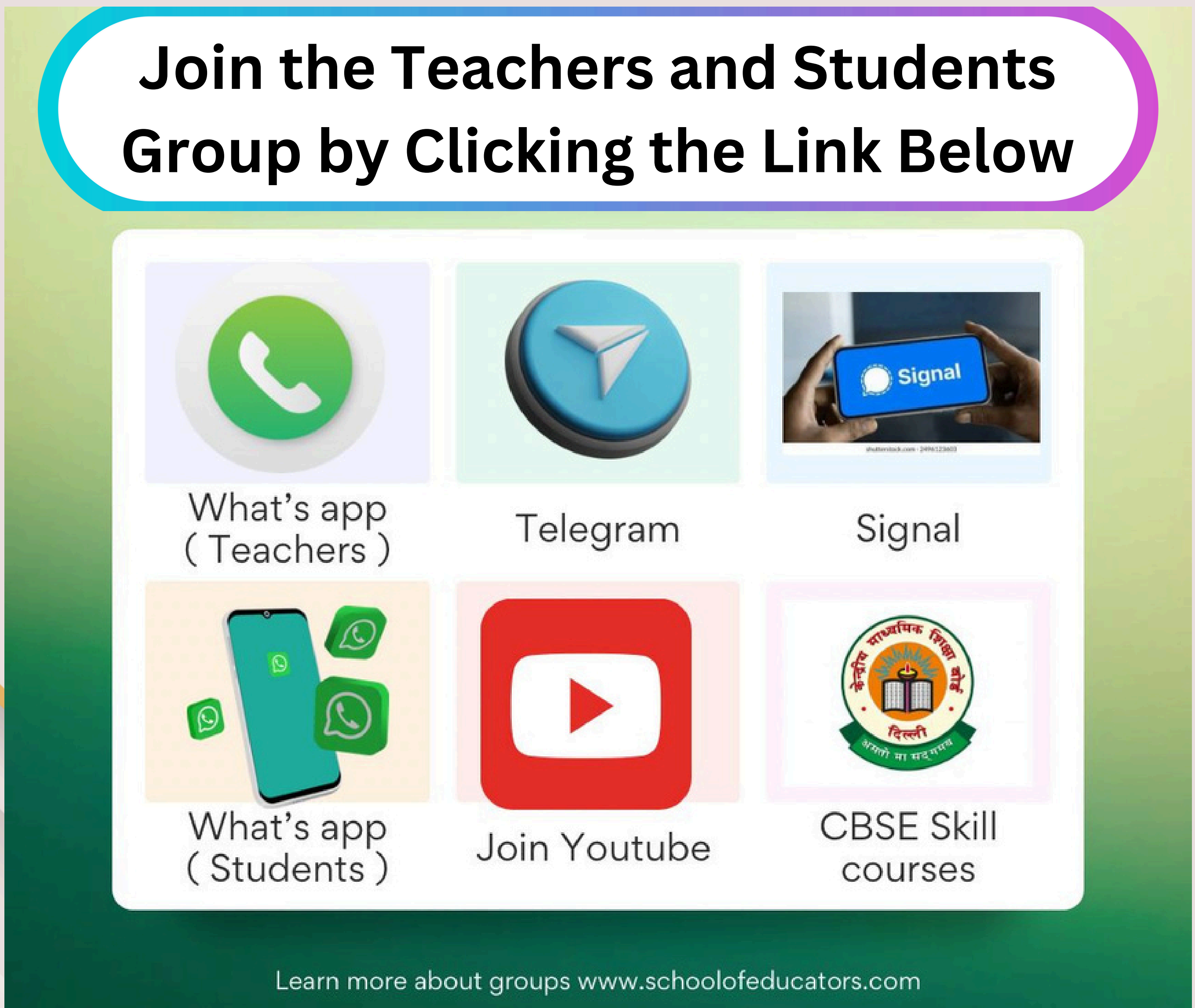


Class – 12 Mathematics

Previous Year Questions

Chapter – 1

Relations And Functions

RELATIONS AND ITS TYPES

Objective Qs (1 mark)

1. Let R be a relation in the set N given by R = {(a, b): $a = b - 2, b > 6$}, then:
(a) $(2,4) \in R$
(b) $(3,8) \in R$
(c) $(6,8) \in R$
(d) $(8,7) \in R$

[CBSE Term-1 SQP 2021]

2. A = {1,2,3,4}. A relation R in the set A is given by R = {(1,1), (2,3), (3,2), (4,3), (3,4)}, then relation?
is:
(a) reflexive
(b) symmetric
(c) transitive
(d) equivalence

[Delhi Gov. Term-1 SQP 2021]

3. Let set X = {1,2,3} and a relation R is defined in X as R = {(1,3), (2,2), (3,2)}, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are:
(a) {(1,1), (2,3), (1,4)}
(b) {(5,3), (3,1), (1,2)}
(c) {(1,1), (3,3), (3,1), (2,3)}
(d) {(1,1), (3,3), (3,1), (1,2)}

[CBSE Term-1 2021]

4. If R = (x, y): $x, y \in Z$, $x^2 + x^2 \leq 4$ is a relation in set ?. Then domain of R is:
(a) {0,1,2}
(b) {−2, −1,0,1,2}
(c) [0, −1, −2]
(d) {−1,0,1}

[CBSE Term-1 2021]

5. A relation R is defined on N. Which of the following is the reflexive relation?
(a) R = {(x, y): $x > y$; $x, y \in N$}
(b) R = {(x, y): $x + y = 10$; $x, y \in N$}
(c) R = {(x, y): xy is the square number; $x, y \in N$}
(d) R = {(x, y): $x + 4y = 10$; $x, y \in N$}

[CBSE Term-1 2021]

6. Let R_+ denote the set of all non-negative real numbers. Then the function
$f : R_+ \rightarrow R+$ defined as $f(x) = x^2 + 1$ is :
(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

(2024)

7. The number of equivalence relations in the set {1,2,3} containing the elements (1,2) and (2,1) is:
(a) 0
(b) 1
(c) 2
(d) 3

[CBSE Term-1 2021]

8. A relation R is defined on Z as: aRb if and only if $a^2 - 7ab^2 + 6b^2 = 0$
Then, R is:
(a) reflexive and symmetric
(b) symmetric but not reflexive
(c) transitive but not reflexive
(d) reflexive but not symmetric

[CBSE Term-1 2021]

9. A function $f : R_+ \to R$ (where R+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is :
(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

(2024)

10. If a matrix has 36 elements, the number of possible orders it can have, is :
(a) 13
(b) 3
(c) 5
(d) 9

(2024)

Very Short & Short Qs (1 - 3 marks)

11. Let R be the equivalence relation on the set Z of integers given by $R = \{(a, b): 2 \text{ divides } a - b\}$. Write the equivalence class {0}.

[CBSE 2021]

12. Check if the relation R in the set R of real numbers defined as $R = \{(a, b): a < b\}$ is:
(a) symmetric;
(b) transitive

[CBSE 2020]

13. Let $R = \{x \in Z: 0 \leq x \leq 12\}$. Show that $R = \{(a, b): a, b \in A, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also, write the equivalence class [2].

[CBSE 2018]

14. If $R = \{(x, y): 2x + y = 8\}$ is a relation on N, write the range of R.

[CBSE 2014]

15. Let $R = a, a^3$: a is a prime number less than 5} be a relation. Find the range of R.

[CBSE 2014]

16. Let R be a relation defined on the set of natural numbers N as $R = \{(x, y): x \in N, y \in N$ and $2x + y = 24\}$. Then, find the domain and range of the relation ?.
Also, find whether ? is an equivalence relation or not.

[CBSE 2014]

17. A relation R on set A = {1, 2, 3, 4, 5} is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive,symmetric and transitive.

(2024)

Long Qs (4 - 5 marks)

18. Let N be the set of all natural numbers and R be a relation on $N \times N$ defined by $(a, b)R(c, d) \Leftrightarrow ad = bc$ for all $(a, b),(c, d) \in N \times N$. Show that R is an equivalence relation on $N \times N$. Also, find the equivalence class of (2,6), i.e., [(2,6)].

[CBSE SQP 2023]

19. Given a non-empty set X, define the relation R in P(X) as follows: For $A,B \in P(X)$, $(A, B) \in R$ if $A \subset B$. Prove that R is reflexive, transitive and not symmetric.

[CBSE SQP 2022]

20. Define the relation R in the set $N \times N$ as follows: For $(a, b),(c, d) \in N \times N$, $(a, b)R(c, d)$ if $ad = bc$. Prove that R is an equivalence relation in $N \times N$.

[CBSE SQP 2022]

21. Show that the relation S in the set $A = [x \in Z : 0 \leq X \leq 12]$ given by $S = [(a, b): a, b \in Z, |a - b|$ is divisible by 3] is an equivalence relation.

[CBSE 2019]

22. Let N denote the set of all natural numbers and R be a relation on $N \times N$ defined by $(a, b)R(c, d)$ if $ad(b+c) = bc(a+d)$. Prove that ? is an equivalence relation.

[CBSE 2015]

23. Show that the relation R in the set A = {1,2,3,4,5} given by $R = \{(a, b): |a - b|$ is divisible by 2} is an equivalence relation. Write all the equivalence classes of R.

[CBSE 2015]

24. Students of a school are taken to a railway museum to learn about railway heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by
$R = \{l_2, l_2$: l1 is parallel to $l_2)\}$
On the basis of the above information, answer thefollowing questions
(i) Find whether the relation R is Symmetric or not.
(ii) Find whether the relation R is transitiveor not.
(iii) If one of the rail lines of the railway track is represented by the equations y = 3x+2, then find the set of rail lines in R related to it. (2024)

FUNCTIONS AND ITS TYPES

Objective Qs (1 mark)

25. Let A = {1,2,3, …, n} and B = {a, b}. Then the number of surjections from A to B is:
(a) ${}^nP_2$
(b) $2^n - 2$
(c) $2^n - 1$
(d) None of these

[Delhi Gov. SQP 2022]

26. Let X = x^2 : X ∈ N and the function F: N → X is defined by f(x) = x^2 , x ∈ N. Then this function is:
(a) injective only
(b) not bijective
(c) surjective
(d) bijective

[CBSE Term-1 2021]

27. A function f: R → R defined by f(x) = 2 + x^2 is:
(a) not one-one
(b) one-one
(c) not onto
(d) neither one-one nor onto

[CBSE Term-1 2021]

28. The function f: R → R defined by f(x) = 4 + 3cos x is:
(a) bijective
(b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto

[CBSE Term-1 2021]

29. The number of functions defined from {1,2, 3,4,5} → {a,b} which are one-one is:
(a) 5
(b) 3
(c) 2
(d) 0

[CBSE Term-1 2021]

30. Let f: R → R be defined by f(x)= $\frac{1}{x}$ $x \epsilon R$. Then f is:
(a) one-one
(b) onto
(c) bijective
(d) f is not defined

[CBSE Term-1 2021]

31. Assertion (A): The relation f: {1,2,3,4} → {x,y,z,p} defined by f = {(1, x), (2, y), (3, z)} is a bijective function.
Reason (R): The function f: {1,2,3} → {x,y,z,p} such that f = {(1, x), (2, y), (3, z)} is one-one.

[CBSE SQP 2023]

Very Short & Short Qs (1 - 3 marks)

32. Let f: N → N be defined as:

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in N.$$

Find whether the function f is bijective or not.

[Delhi Gov. SQP 2022]

33. Let f: X → Y be a function. Define a relation R on X given by R = {(a, b): f(a) = f(b)}. Show that R is an equivalence relation on X.

[Delhi Gov. SQP 2022]

34. Prove that the function f is surjective, where f: N → N such that

$$f(n) = \begin{cases} \frac{n+1}{2}, \text{if n is odd} \\ n^2, \text{if n is even} \end{cases}$$

Is the function injective? Justify your answer.

[CBSE SQP 2022]

35. Let A = {1,2,3}, B = {4,5,6,7} and f = {(1,4), (2,5), (3,6)} be a function from A to B. State whether' f' is one-one or not.

[CBSE 2014]

Long Qs (4 - 5 marks)

36. Show that the function f:R → {x ∈ R : −1 < x < 1} defined by f(x) = $\frac{x}{1+|x|}$, $x \in R$ is one-one and onto function.

[CBSE SQP 2023]

37. A function f: [− 4,4] → [0,4] is given by f(x) = $\sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which f(a) $= \sqrt{7}$.

[CBSE 2023]

38. Show that the function f: (− ∞, 0) → (− 1,0) and f(x) = $\frac{x}{1+|x|}$, $x \in (-\infty, 0)$ is one-one and onto.

[CBSE 2020]

39. (a) Show that a function f : R → R defined by f(x) = $\frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function f : R → A becomes an onto function.

(2024)

OR

(b) A relation R is defined on N ´ N (where N is the set of natural numbers) as :
(a, b) R (c, d) g a – c = b – d
Show that R is an equivalence relation.

(2024)

40. Let A= R – { 5} a nd B = R – { 1}. Consider the functionf: A→B,definedbyf(x)= $\frac{x-3}{x-5}$. Showthatfisone-oneandonto.

(2024)

41. Check whether the relation Sin the set of real numbers R defined by S = {(a, b): where a$\sqrt{}$b+ 2 is an irrational number} is reflexive, symmetri cortransitive.

(2024)

Class – 12 Mathematics

PYQ Solutions

Chapter – 1

Relations And Functions

1. RELATIONS AND ITS TYPES

1. (c) $(6,8) \in R$
   Explanation: Given,
   $a = b - 2$ and $b > 6$
   $\Rightarrow (6,8) \in R$

2. (b) symmetric
   Explanation: We have,
   $R = \{(1,1), (2,3), (3,2), (4,3), (3,4)\}$
   $\because (2,2), (3,3), (4,4) \notin R$
   $\therefore$ R is not reflexive.
   For $(2,3) \in R$, we have $(3,2) \in R$
   Similarly for $(4,3) \in R$, we have $(3,4) \in R$.
   $\therefore$ R is symmetric.
   For $(2,3) \in R$ and $(3,2) \in R$, we have $(2,2) \notin R$
   $\therefore$ R is not transitive
   $\because$ R is not reflexive and transitive so it is not an equivalence relation.

   NOTE:
   A relation R in a set A is called
   (i) reflexive, if $(a, a) \in R$, for every $a \in A$,
   (ii) symmetric, if $(a_1, a_2) \in R$ implies that $a_2, a_1 \in R$, For all $a_1, a_2 \in A$.
   (iii) transitive, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $a_1, a_3 \in R$ for all $a_1, a_2, a_3 \in R$.

3. (c) $\{(1,1), (3,3), (3,1), (2,3)\}$
   Explanation: The ordered pairs to be added in R are:
   (1,1), (3,3) {needed to make R reflexive}
   (3,1), (2,3) {needed to make R symmetric}
   So, $\{(1,1)(3,3), (3,1), (2,3)\}$ are required.

4. (b) $\{-2, -1, 0, 1, 2\}$
   Explanation: Given,
   $x^2 + y^2 \leq 4$
   $\Rightarrow y^2 \leq 4 - x^2$
   $\Rightarrow y \leq \sqrt{4 - x^2}$
   For domain: $4 - x^2 \geq 0$
   $\Rightarrow x^2 \leq 4$
   $\therefore -2 \leq x \leq 2$
   So, Domain $= \{-2, -1, 0, 1, 2\}$

5. (c) $R = \{(x, y) : xy$ is the square number; $x, y \in N\}$ Explanation: When $x \in N$, x^2 is a square number
   So, $(x, x) \in R$ for all $x \in N$.
   Therefore, R is reflexive.

6. (A) one-one but not onto

7. (c) 2
   Explanation: Total possible pairs = (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)
   Reflexive means (a.a) should be in relation.
   So, (1,1), (2,2), (3,3) should be in relation.
   Symmetric means if (a, b) is in relation, then (b, a) should be in relation.
   So, since (1,2) is in relation, (2,1) should also be in relation.
   Transitive means if (a, b) is in relation and (b, c) is in relation, then (a, c) is in relation.
   So, if (1,2) is in relation and (2,1) is in relation, then (1,1) should be in relation.
   Relation R_1 = {(1,2), (2,1), (1,1), (2,2), (3,3)}
   Total possible pairs = (1,1), (1,2), (1,3), (2,1), (2, 2), (2,3), (3,1), (3,2), (3,3)
   So, smallest relation is R_1 = {(1,2), (2,1), (1,1), (2,2), (3,3)}.
   If we add (2,3)
   then we have to add (3,2) also, as it is symmetric but, as (1,2) and (2,3) are there, we need to add (1,3) also, as it is transitive.
   As we are adding (1,3) we should add (3,1) also, as it is symmetric.
   Relation R_2 = {(1,2), (2,1), (1,1), (2,2), (3,3), (2,3), (3,2), (1,3), (3,1)}
   Hence, only two possible relations are there which are equivalence.

8. (d) reflexive but not symmetric
   Explanation: We have,
   $a^2 - 7aa + 6a^2 = 7a^2 - 7a^2 = 0$
   Therefore, a Ra for all a in Z
   So R is reflexive
   Let aRb, then $a^2 - 7ab + 6b^2 = 0$
   Consider $b^2 - 7ba + 6a^2 = b^2 - a^2 + 6b^2 + 6a^2 = 5a^2 - 5b^2$
   So, R is not symmetric.

9. (a) One-one but not onto

10. (d) 9

11. R = {(a, b): 2 divides (a − b)}
   ⇒ (a − b) is a multiple of 2.
   To find equivalence class 0, put b = 0
   So, (a − 0) is a multiple of 2
   ⇒ a is a multiple of 2
   So, in set Z of integers, all the multiple of 2 will come in equivalence class {0}
   Hence, equivalence class {0} = {2x}
   where x = integer (Z).
   NOTE
   → An equivalence class of a is denoted as [a] = {x ∈ A : (a, x) ∈ R}. This comprises all of A 's elements related to letter ' a '.

12. $R = \{(a, b): a < b\}$

(A) Checking for symmetric, if $(a, b) \in R$ such that $a < b$ then, $(b, a) \in R$ not possible $(2,3) \in R$ but, $(3,2) \in R$ $\therefore$ Relation R is not symmetric.

(B) Checking for transitive,

if $(a, b) \in R$ and $(b, c) \in R$ such that $a < b$ and $b < c$ then clearly, $a < c$ i.e., $(a, c) \in R$

$\therefore$ Relation R is transitive.

13. For reflexive:

Let $a \in Z$, then $|a - a| = 0$, which is divisible by 4. So, $(a, a) \in R$. Thus, R is a reflexive .

For symmetric:

Let $a, b \in Z$ such that $|a - b|$ is divisible by 4. Then $|b - a|$ is also divisible by 4

$[\because |b - a| = |a - b|]$

So, $(a, b) \in R \Rightarrow (b, a) \in R$.

Thus, R is a symmetric relation.

For transitive:

Let $a,b,c \in Z$ and $(a, b) \in R$ and $(b, c) \in R$.

Since $(a, b) \in R$ and $(b, c) \in R$

Therefore, $|a - b| = 4k$ for some $k \in z$

$\Rightarrow (a - b) = \pm 4k$

and $|b- c| = 4l$ for some $l \in Z$

$\Rightarrow (b - c) =\pm 4l$

Now, $(a - c) = (a - b) + (b - c)$

$=\pm 4k \pm 4l$

$= 4(\pm k \pm l)$, which is divisible by 4.

So, $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a, c) \in R$. Thus, R is transitive.

Since, R is reflexive, symmetric and transitive, hence, it is an equivalence relation.

Set of elements related to 1 is $\{1,5,9\}$

Set of elements related to 2 is $\{2,6,10\}$

So, equivalence class of [2] is $\{2,6,10\}$.

14. $R = \{(x, y): 2x + y = 8\}$ is a relation on N.

Therefore, $R = \{(3,2), (2,4), (1,6)\}$

So, Range $= \{2,4,6\}$.

15. Relation $R = a, a^3$: a is a prime number less than 5}.

Therefore, $R = \{(2,8), (3,27)\}$,

$\Rightarrow$ Range $= \{8,27\}$

16. We have,

$2x + y = 24$

$\Rightarrow y = 24 - 2x$

For $x = 1$, $y = 24 - 2 \times 1 = 24 - 2 = 22$

For $x = 2$, $y = 24 - 2 \times 2 = 24 - 4 = 20$

For $x = 3$, $y = 24 - 2 \times 3 = 24 - 6 = 18$

For $x= 4$, $y= 24 - 2 \times 4 = 24 - 8 = 16$

For $x = 5$, $y = 24 - 2 \times 5 = 24 - 10 = 14$
For $x = 6$, $y = 24 - 2 \times 6 = 24 - 12 = 12$
For $x = 7$, $y = 24 - 2 \times 7 = 24 - 14 = 10$
For $x = 8$, $y = 24 - 2 \times 8 = 24 - 16 = 8$
For $x = 9$, $y = 24 - 2 \times 9 = 24 - 18 = 6$
For $x = 10$, $y = 24 - 2 \times 10 = 24 - 20 = 4$
For $x = 11$, $y = 24 - 2 \times 11 = 24 - 22 = 2$
In ordered pair, R = {(1,22), (2,20), (3,18),
(4,16), (5,14), (6,12), (7,10), (8,8), (9,6), (10,4), (11,2)}
∴ Domain of R = {1,2,3,4,5,6,7,8,9,10,11}
Range of R = {22,20,18,16,14,12,10,8,6,4,2}
Here, $1 \in N$ but $(1,1) \notin R$, hence R is not reflexive.
Hence, R is not an equivalence relation.

17. 1Set A={1,2,3,4,5}
Relation R on set A is defined as : $R=\{(x,y): |x^2-y^2| <8\}$
$R=\{(x,y): |x^2-y^2| < 8\}$
We need to check whether the relation RRR is reflexive, symmetric, and transitive.
Reflexive:
- A relation R on set A is reflexive if $(a,a)\in R$ for all $a \in A$.
- For each $a \in A$, we need to check if $|a^2-a^2| < 8$.
- $|a^2-a^2| = 0$, which is always less than 8.
- Therefore, R is reflexive .

Symmetric:
- A relation R on set A is symmetric if $(a,b)\in R \Longrightarrow (b,a)\in R$ for all $a,b\in A$.
- If $(a,b)\in R$, then $|a^2-b^2| <8$ . .
- $|a^2-b^2| = |b^2-a^2|$ which means $(b,a)\in R$.
- Therefore, R is symmetric.

Transitive:
- A relation R on set A is transitive if $(a,b)\in R$ and $(b,c)\in R \Longrightarrow (a,c)\in R$ for all $a,b,c \in A$.
- Suppose $(a,b) \in R$ and $(b,c) \in R$, i.e., $|a^2-b^2| < 8$ and $|a^2-c^2| < 8$.
- This does not necessarily imply that $|a^2-c^2| <8$.
- For example, let a=1, b=3, and c=5 . □
- $|1^2-3^2| = |1-9| =8|$ which is not less than 8.
- Therefore, R is not transitive.

NOTE:
A relation is said to be equivalence if it is reflexive, symmetric and transitive.

18. Let (a, b) be an arbitrary element of $N \times N$. Then, $(a, b) \in N \times N$ and $a, b \in N$
We have, ab = ba; (As $a, b \in N$ and multiplication is commutative on N)
⇒ (a, b)R(a, b), according to the definition of the relation R on $N \times N$.
Thus, (a, b)R(a, b), $\forall (a, b) \in N \times N$.
So, R is reflexive relation on $N \times N$.

Let (a, b), (c, d) be arbitrary elements of N × N such that (a, b)R(c, d).
Then, (a, b)R(c, d)
⇒ ad = bc
⇒ bc = ad
(changing LHS and RHS)
⇒ cb = da; (As a, b, c, d ∈ N and multiplication is commutative on N)
⇒ (c, d)R(a, b); according to the definition of the relation R on N × N
Thus, (a, b)R(c, d)
⇒ (c, d)R(a, b)
So, R is symmetric relation on N × N.
Let (a, b)(c, d), (e, f) be arbitrary elements of N × N such that
(a, b)R(c, d) and (c, d)R(e, f).
Then (a, b)R(c, d)
⇒ ab = bc
and (c, d)R(e, f)
⇒ cf =de
⇒ (ad)(cf) = (bc)(de)
⇒ af = be
⇒ (a, b)R(e, f); (according to the definition of the relation R on N × N)
Thus, (a, b)R(c, d) and (c, d)R(e, f)
⇒ (a, b)R(e, f)
So, R is transitive relation on N × N.
As the relation R is reflexive, symmetric and transitive so, it is equivalence relation on N× N.
[(2,6)] = {(x, y) ∈ N × N: (x, y)R(2,6)}
= {(x, y) ∈ N × N: 3x = y}
= {(x, 3x): x ∈ N}
= {(1,3), (2,6), (3,9), ……. . }

19. Let A ∈ P(X). Then A ⊂ A
⇒ (A, A) ∈ R
Hence, R is reflexive.
Let A, B, C, ∈ P(X) such that
(A, B), (B, C) ∈ R
⇒ A ⊂ B, B, ⊂ C
⇒ A ⊂ C
⇒ (A, C) ∈R
Hence, R is transitive.
ϕ, X ∈ P(X) such that ? ⊂ X. Hence, (ϕ, X) ∈ R. But X ⊄ ϕ
which implies that (X, ϕ) ∉ R.
Thus, R is not symmetric.

20. Let (a, b) ∈ N × N. Then we have,
ab = ba (by commutative property of multiplication of natural numbers)
⇒ (a, b)R(a, b)

Hence, R is reflexive.
Let (a, b), (c, d) $\in$ N × N such that (a, b)R(c, d).
Then ad = bc
$\Rightarrow$ cb = ba (by commutative property of multiplication of natural numbers
$\Rightarrow$ (a, b)R(a, b)
Hence, R is symmetric.
Let (a, b)(c, d), (e, f) $\in$ N × N such that
(a, b)R(c, d) and (c, d)R(e, f).
Then, ad = bc, cf = de
$\Rightarrow$ adcf = bcde
$\Rightarrow$ af = be
$\Rightarrow$ (a, b)R(e, f)
Hence, R is transitive.
Since, R is reflexive, symmetric and transitive, R is an equivalence relation on N × N.

21. A = {x $\in$ Z: $0 \le x \le 12$} = {1,2,3,4,5,6,7,8,9, 10,11,12}
R = {(a, b): |a − b| is divisible by 3}
For any element a $\in$ A, we have (a, a) $\in$ R as |a − a| = 0 is divisible by 3.
$\therefore$ R is reflexive.
Now, let (a, b) $\in$ R $\Rightarrow$ |a − b| is divisible 3.
$\Rightarrow$ | b − a)| = |a − b | is divisible by 3
$\Rightarrow$ (b, a) $\in$ R
$\therefore$ R is symmetric.
Now, let (a, b), (b, c) $\in$ R.
$\Rightarrow$ |a − b| is divisible by 3 and |b − c| is divisible by 3.
$\Rightarrow$± (a − b) is divisible by 3 and ±(b − c) is divisible by 3.
$\Rightarrow$ (a − c) = (a − b) + (b − c) is divisible by 3.
and −(a − c) =− {(a − b) + (b − c)} is divisible by 3.
$\Rightarrow$ |a − c| is divisible by 3.
$\Rightarrow$ (a, c) $\in$ R
$\therefore$ R is transitive.
Hence, R is an equivalence relation.

22. Reflexive: Let (a, b) $\in$ N × N.
$\because$ ab(b + a) = ba(a + b)
$\Rightarrow$ (a, b)R(a, b)
$\Rightarrow$ R is reflexive.
Symmetric: For (a, b), (c, d) $\in$ N × N such that (a, b)R(c, d)).
$\therefore$ad(b + c) = bc(a + d)
or, bc(a + d) = ad(b + c)
or, cb(d + a) = da(c + b)
$\Rightarrow$ (c, d)R(a, b)
$\Rightarrow$ R is symmetric.
Transitive: Let (a, b), (c, d)), (e, f) $\in$ N × N such that (a, b)R(c, d)) and (c, d)R(e, f).
$\therefore$ ad(b + c) = bc(a + d)
and cf(d+ e) = de(c + f)
$\Rightarrow$ aba + adc = bca+ bcd….(i)
and cdf + cfe = dec + def ….(ii)

Multiplying (i) by ef and (ii) by ab and then adding them, we get
adbef+ adcef + cfdab + cfeab = bcaef + bcdef +decab + defab

$\Rightarrow$ adcef + adcfb = bcdea + bcdef

$\Rightarrow$ adcf (e + b) = bcde(a + f)

$\Rightarrow$ af(b + e) = be(a + f)

$\Rightarrow$ (a, b)R(e, f)

$\Rightarrow$ R is transitive.

Hence, R is an equivalence relation.

23. Given R = {(a, b): |a − b| is divisible by 2}
and A = {1,2,3,4,5}
R = {(1,1), (2,2), (3,3), (4,4), (5,5)(1,3), (1,5), (2,4), (3,5), (3,1), (5,1), (4,2), (5,3)}
Reflexive:

$\forall$a $\in$ A, (a, a) $\in$ R,

$\therefore$ R is reflexive

$\therefore$ [As {(1,1), (2,2), (3,3), (5,5)} $\in$ R]

Symmetric

$\forall$(a, b) $\in$ R, (b, a) $\in$ R,

$\therefore$ R is symmetric.

[As {(1,3), (1,5), (2,4), (3,5), (3,1), (5,1), (4,2), (5,3)} $\in$ R]

Transitive

$\forall$(a, b), (b, c) $\in$ R, (a, c) $\in$ R

$\therefore$ R is transitive.

[As (1,3), (3,1) $\in$ R $\Rightarrow$ (1,1) $\in$ R and similarly others]

$\therefore$ R is an equivalence relation.

Equivalence classes are
and

[1] = {1,3,5}
[2] = {2,4}
[3] = {1,3,5}
[4] = {2,4}
[5] = {1,3,5}

24. **Solutions:**

1. **Symmetric Relation**:
A relation R on a set A is symmetric if for every $(a, b) \in R$, $(b, a) \in R$.
Checking Symmetry:
   - Given $(l_1, l_2) \in R$ l implies 1 is parallel to l_2.

   - Parallelism is symmetric: If is parallel to , then $l2$ is also parallel to $l1$.
   - Hence, $(l_2, l_1) \in R$ whenever $(l_1, l_2) \in R$.

   Therefore, the relation R is symmetric.

2. **Transitive Relation**:
A relation R on a set A is transitive if for every $(a, b) \in R$ and , $(a, c) \in R$.

Checking Transitivity:

- Given (l_1, l_2)and (l_2, l_3), this implies l_1 is parallel to l_2 and l_2 is parallel to l_3.
- Parallelism is transitive: If l_1 is parallel to l_2 and l_2 is parallel to l_3, then l_1 is also
- parallel to l_3.
- Hence, $(l_1, l_3) \in$ R.

Therefore, the relation R is transitive.

3. **Set of Rail Lines Related to the Given Line**:
   If one of the rail lines on the railway track is represented by the equation 2, we need to find the set of rail lines in R related to it.

   Equation of the Given Line:
   - The given line is y = 3x + 2.

   Finding Related Lines:

   - Lines related to this one are parallel lines with the same slope, which is .
   - The general equation of lines parallel to $y = 3x + 2$ is $y = 3x + c$, where c is any real number.

   Therefore, the set of rail lines in R related to $y = 3x + 2$ is:

$$\{y = 3x + c \mid c \in \mathrm{R}\}$$

2. FUNCTIONS AND ITS TYPES

25. (b) $2^n - 2$
    Explanation: Given that, A = {1,2,3, …n} and B = {a, b}
    If function is surjective then its range must be set B = {a, b}
    Now number of onto functions = Number of ways ' n' distinct objects can be distributed in two boxes ' a ' and ' b ' in such a way that no box remains empty.
    Now for each object there are two options, either it is put in box 'a' or in box 'b'
    So total number of ways of ' n ' different objects put in two boxes = $2 \times 2 \times 2 \ldots$n times = 2^n
    But in one case all the objects are put in box ' a ' and in one case all the objects are put in box ‘b’.So,
    number of surjective functions = $2^n - 2$

26. (d) bijective
    Explanation: Given, $f(x) = x^2$
    For injective:
    Let $f(x_1) = f(x_2)$
    $\Rightarrow x_1^2 = x_2^2$
    $\Rightarrow x_1 = x_2$
    ($\because$ - ve rejected)
    So, it is injective.
    For subjective:

    Range = X
    Co-domain = X
    So, range = co-domain

So, it is subjective.
Hence, it is bijective.

27. (d) neither one-one nor onto

Explanation:

For one-one: $f(x_1) = f(x_2)$

$\Rightarrow 2 + x_1^2 = 2 + x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2 \because x_1, x_2 \in R$

So, it is not one-one.
For onto:
Range = Positive real numbers

Co-domain = R

$\therefore$ Range $\neq$ co-domain

So, it is not onto.
Hence, f(x) is neither one-one nor onto.

28. (d) neither one-one nor onto

Explanation: Given: f(x) = 4 + 3cosx

Since, $\cos\frac{\pi}{2} = \cos\left(-\frac{\pi}{2}\right)$

$\Rightarrow 4 + 3\cos\frac{\pi}{2} = 4 + \cos\left(-\frac{\pi}{2}\right)$

$\Rightarrow f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$

But

$$\frac{\pi}{2} \neq -\frac{\pi}{2}$$

So, f is not one-one.
Range of cos x is $[-1,1]$

$\Rightarrow -1 \leq \cos x \leq 1$

$\Rightarrow -3 \leq 3\cos x \leq 3$

$\Rightarrow 1 \leq 4 + 3\cos x \leq 7$

$\Rightarrow 1 \leq f(x) \leq 7$

So, the range of f is [1,7]
Thus, f is not onto.
Hence, f is neither one-one nor onto.

29. (d) 0

Explanation: If we consider one-one function, only two elements of the set {1,2,3,4,5} an have images.

Therefore, there can't be a one-one function from {1,2,3,4,5} → {a, b}

Hence, the number of one-one functions is 0.

30. (d) f is not defined

Explanation: We have, $f(x) = \frac{1}{x}, \forall x \in R$
For x= 0, f(x) is not defined.
Hence, f(x) is not a defined function.

31. (d) (A) is false but (R) is true.

Explanation: Assertion is false. As element 4 has no image under f, so relation f is not a function.

Reason is true. The given function ?: {1, 2,3} → {x, y, z, p} is one - one, as for each a ∈ {1,2,3}, there is a different image in {x, y, z, p} under f.

27.The given function is f: N → N such that

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if n is odd} \\ & for\ all\ \text{n} \in N \\ \frac{n}{2}, & \text{if n is even} \end{cases}$$

32. We shall verify whether f(x) is one-one and onto.

One-One:

From the definition of f(n)

$f(1) = \frac{1+1}{2} = 1$ and $?(2) = \frac{2}{2} = 1$

f(n) is not an one-one function because at two distinct values from domain (N), f(n) has same image.

Onto: For onto function, we check whether

Range of f(n) = Co-domain of f(n)

Now, if n is an odd natural number, then (2n − 1) is also an odd natural number.

Now, $f(2n) - 1 = \frac{2n-1+1}{2} = n \ldots (i)$

Again, if n is an even natural number, then 2n is also an even natural number. Then

$$f(2n) - 1 = \frac{2n}{2} = n \ldots (ii)\#(ii)$$

From equations, (i) and (ii), we observe that for each n (whether even or odd), there exists its pre image in N.

i.e., Range of f(n) = Co-domain of f(n).

Hence, f is onto.

Since, f(n) is onto but not one-one, it is not a bijective function.

33. The given function is f: X → Y and relation on X is R = {(a, b): f(a) = f(b)}

Reflexive:

Since, for every x ∈ X, we have

f(x) = f(x)

⇒ (x, x) ∈R, ∀x ∈ X

Therefore, R is reflexive.

Symmetric:

Let (x, y) ∈ R

Then, f(x) = f(y)

⇒ f(y) = f(x)

⇒ (y, x) ∈ R

Thus, (x, y) ∈ R

⇒ (y, x) ∈ R, ∀x, y ∈ X

Therefore, R is symmetric.

Transitive:

Let x, y, z ∈ X such that

(x, y) ∈ R and (y, z) ∈ R

Then, f(x) = f(y)…(i)

And f(y) = f(z) …. (ii)

From eqs. (i) and (ii), we get

$f(x) = f(z)$
$\Rightarrow (x, z) \in R$

Thus, $(x, y) \in R$ and $(y, Z) \in R$
$\Rightarrow (x, y) \in R, \forall x, y, z \in R$
Therefore, R is transitive.
Since, R is reflexive symmetric and transitive, so it is an equivalence relation.

34. Let $y \in N$ (co-domain). Then $\exists\, 2y \in N$ (domain) such that $f(2y) = \frac{2y}{2} = y$ Hence, f is surjective.
$1,2 \in N$ (domain) such that $f(1) = 1 = f(2)$
Hence, f is not injective.

35. Given that $A = \{1,2,3\}$, $B = \{4,5,6,7\}$ Now, $f: A \rightarrow B$ is defined as $f = \{(1,4), (2,5), (3,6)\}$.
$f(1) = 4$, $f(2) = 5$, $f(3) = 6$, so f is one-one.

36. We have, $f(x) = \begin{cases} \frac{x}{1+x}, & \text{if } x \geq 0 \\ \frac{x}{1-x,}, & \text{if } x < 0 \end{cases}$

Now, we consider the following cases
Case 1: when $x \geq 0$, we have $f(x) = \frac{x}{1+x}$ Injectivity: let $x, y \in R^+ \cup \{0\}$ such that $f(x) = f(y)$, then

$$\Rightarrow \frac{x}{1+x} = \frac{y}{1+x}$$
$$\Rightarrow x + xy = y + xy$$
$$\Rightarrow x = y$$

So, f is injective function.
Surjectivity: when $x \geq 0$, we have $f(x) = \frac{x}{1+x} \geq 0$ and $f(x) = 1 - \frac{x}{1+x} < 1$, as $x \geq 0$

Let $y \in [0,1)$, thus for each $y \in [0,1)$ there exists $x = \frac{y}{1-y} \geq 0$ such that $f(x) = \frac{\frac{y}{1-y}}{1+\frac{y}{1-y}} = y.$

So, f is onto function on $[0, \infty)$ to $[0,1)$.
Case 2: when $x < 0$, we have $f(x) = \frac{x}{1-x}$
Injectivity: Let $x, y \in R^-$ i.e., $x, y < 0$, such that $f(x) = f(y)$, then

$$\Rightarrow \frac{x}{1-x} = \frac{y}{1-y}$$
$$\Rightarrow x - xy = y - xy$$
$$\Rightarrow x = y$$

So, f is injective function
Surjective: $x < 0$,
We have $f(x) = \frac{x}{1-x} < 0$
also, $f(x) = \frac{x}{1-x} = -1 + \frac{x}{1-x} > -1$
$-1 < f(x) < 0$.
Let $y \in (-1,0)$ be an arbitrary real number and there exists $x = \frac{y}{1+y} < 0$ such that,

$$f(x) = f\left(\frac{y}{1+y}\right) = \frac{\frac{y}{1+y}}{1-\frac{y}{1+y}} = y$$

So, for $y \in (-1,0)$, there exists $x = \frac{y}{1+y} < 0$ such that $f(x) = y$.

Hence, f is onto function on $(-\infty, 0)$ to $(-1,0)$.

Case 3:

(Injectivity): Let $x > 0$ and $y < 0$ such that $f(x) = f(y)$

$$\Rightarrow \frac{x}{1+x} = \frac{y}{1-y}$$

$$\Rightarrow x - xy = y + xy$$

$$\Rightarrow x - y = 2xy,$$

Here LHS > 0 but RHS < 0, which is inadmissible. Hence, $f(x) \neq f(y)$ when $x \neq y$.

Hence, f is one-one and onto function.

37. $$f(x) = \sqrt{16 - x^2}$$

for

$$x = 2, f(x) = \sqrt{12}$$

for

$$x = -2, f(x) = \sqrt{12}$$

Since, for x = 2 and -2, the function has same image

∴ The given function is not one-one.

Let ? $\in [0,4]$

∴ $y \geq 0$

$$y = \sqrt{16 - x^2}$$

$$y^2 = 16 - x^2$$

$$x = \sqrt{16 - y^2}$$

For $x \in R$, $16 - y^2 \geq 0$

$$(4 - y)(4 + y) \geq 0$$

$\Rightarrow$ Either $(4 - y) \geq 0$ and $(4 + y) \geq 0$ or $(4 - y) < 0$ and $(4 + y) < 0$.

∴ For every $y \in [0,4]$ $\exists$ $x \in [-4,4]$ such that $y = f(x)$

∴ The given function is onto

$$f(a) = \sqrt{12}$$

$$\sqrt{16 - y^2} = \sqrt{7}$$

Squaring on both sides:

$$16 - a^2 = 7$$

$$a^2 = 9$$

$$a = \pm 3$$

38. Given,

$$f(x) = \frac{x}{1+|x|}, x \in (-\infty, 0)$$
$$= \frac{x}{1-x}$$
$$[\because x \in (-\infty, 0), |x| = -x]$$

For one-one:

Let $f(x_1) = f(x_2)$, $x_1, x_2 \in (-\infty, 0)$

$$\Rightarrow \frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$$
$$\Rightarrow x_1(1- x_2) = x_2(1-x_1)$$
$$\Rightarrow x_1 - x_1x_2 = x_2 - x_1x_2$$
$$\Rightarrow x_1 = x_2$$

Hence, if $f(x_1) = f(x_2)$, then $x_1 = x_2$

$\therefore$ f is one-one

For onto:

Let $f(x) = y$

$\Rightarrow y = \frac{x}{1-x}$

$\Rightarrow y(1 - x) = x$

$\Rightarrow y - xy = x$

$\Rightarrow x + xy = y$

$\Rightarrow x(1+y) = y$

$\Rightarrow x = \frac{y}{1+y}$

So, x is defined for all values of y.

$\therefore$ f is onto.

39. (a)

Given the function:

$f : R \to R$ defined by $f(x) = \frac{2x}{1+x^2}$

1. **Show that $f(x)$ is neither one-one nor onto:**

(a) $f(x)$ is not one-one:

To show that $f(x)$ is not one-one (injective), we need to find two distinct values x_1 and x_2 *Such that* $f(x_1) = f(x_2)$.

Suppose : $f(x_1) = f(x_2)$

$$\frac{2x_1}{1+x_1^2} = \frac{2x_2}{1+x_2^2}$$

Cross-multiplying gives:

$$2x_1(1+x_2^2) = 2x_2(1+x_1^2)$$
$$x_1 + x_1x_2^2 = x_2 + x_2x_1^2$$

Rearrange the terms:

$$x_1 - x_2 = x_2x_1^2 - x_1x_2^2$$

Factor out $x_1 - x_2$:

$$x_1 - x_2 = x_2 x_1(x_1 - x_2)$$

If $x_1 \neq x_2$ then:

$$1 = x_2 x_1$$

Therefore, $x_1 = \frac{1}{x^2}$

So, for example, $f(1) = \frac{2.1}{1+1^2} = 1$ and $f(-1) = \frac{2.(-1)}{1+(-1^2)} = -1$. This demonstrates that the function f is not one-one because different values of x can produce the same value of $f(x)$

(b) $f(x)$ is not onto:

To show that $f(x)$ is not onto (surjective), we need to show that there is some y $\in$R such that there is no $x \in$ R for which $f(x) = y$.

The function $f(x)$ is defined as:

$$f(x) = \frac{2x}{1 + x^2}$$

Let's find the range of $f(x)$:

The maximum value of $f(x)$ occurs when $\frac{d}{dx}\left(\frac{2x}{1+x^2}\right) = 0$

Using calculus, we find:

$$\frac{d}{dx}\left(\frac{2x}{1+x^2}\right) = \frac{2(1+x^2) - 2x.2x}{(1+x^2)^2} = \frac{2 - 2x^2}{(1+x^2)^2}$$

Setting the derivative equal to zero:

$$2 - 2x^2 = 0$$

$$x^2 = 1 \Longrightarrow x = \pm 1$$

The values at $x = 1$ and $x = -1$

$$f(1) = 1 \text{ and } f(-1) = -1$$

We can observe that as $x \to \infty$ or $x \to -\infty$, $f(x) \to 0$

OR

(b)

Given the relation R defined on N × N (where N is the set of natural numbers) as:

$(a, b)R(c, d) \Longleftrightarrow a - c = b - d$

We need to show that R is an equivalence relation.

An equivalence relation must satisfy three properties: reflexivity, symmetry, and transitivity.

1. **Reflexivity:**

A relation R on a set S is reflexive if every element is related to itself.

For R to be reflexive:

$(a, b)R(a, b)$ for all $(a, b) \in$ N × N

Check:

$$a - a = b - b \Longrightarrow 0 = 0$$

Since 0 = 0 is always true, R is reflexive

Symmetry:

A relation R on a set S is symmetric if whenever a is related to b, b is related to a.

For R to be symmetric:

If $(a, b)R(c, d)$, then $(c, d)R(a, b)$

Check:

$(a, b)R(c, d) \Longrightarrow a - c = b - d$

Then $(c, d)R(a, b) \Longrightarrow c - a = d - b$

Since $a - c = b - d$ implies $c - a = d - b$ (multiplying both sides by -1), R is symmetric.

3. **Transitivity:**

A relation R on a set S is transitive if whenever a is related to b and b is related to c, a is related to c.

For R to be transitive:

If $(a, b)R(c, d)$ and $(c, d)R(e, f)$, then $(a, b)R(e, f)$

Check:

$$(a, b)R(c, d) \Longrightarrow a - c = b - d$$
$$(c, d)R(e, f) \Longrightarrow c - e = d - f$$

Adding these equations:

$$(a - c) + (c - e) = (b - d) + (d - f)$$
$$a - e = b - f$$

Therefore, which means R is transitive

40. To show that the function $f : A \rightarrow B$ defined by $f(x) = \frac{x-3}{x-5}$, where $A = R - \{5\}$ and $B = R - \{1\}$ is one-one and onto, we need to prove both injectivity and surjectivity.

Injectivity (One-One)

A function f is injective if $f(a) = f(b)$ implies $a = b$ for all .

Let :$f(a) = f(b)$

$$\frac{a-3}{a-5} = \frac{b-3}{b-5}$$

Cross-multiplying

$(a - 3)(b - 5) = (b - 3)(a - 5)$

Expanding both sides:

$$ab - 5a - 3b + 15 = ab - 3a - 5b + 15$$

Simplifying:

$$-5a - 3b = -3a - 5b$$
$$-5a + 5b = -3a + 3b$$
$$-2a = -2b$$
$$a = b$$

Thus, f is injective.

Surjectivity (Onto)

A function f is surjective if for every , there exists an $x \in A$ such that .

Let $y \in B$:

$$y = \frac{x - 3}{x - 5}$$

Solving for x:

$y(x - 5) = x - 3$
$yx - 5y = x - 3$
$yx - x = 5y - 3$
$x(y - 1) = 5y - 3$
$x = \frac{5y-3}{y-1}$

We need to ensure $x \neq 5$
:

$$\frac{5y-3}{y-1} \neq 5$$

Check if $x = 5$ leads to any contradiction:

$5 = \frac{5y-3}{y-1}$
$5(y - 1) = 5y - 3$
$5y - 5 = 5y - 3$
$-5 \neq -3$

Thus, x = $\frac{5y-3}{y-1}$ is always valid, and since $y \neq 1$, $x \in A$

Therefore, f is surjective

Conclusion

The function $f(x) = \frac{x-3}{x-5}$ is both one-one and onto.

41. **1. Reflexivity:**

- A relation is reflexive if every element is related to itself. That is, for all $a \in R$, $(a, a) \in S$
- For (a, a) to be in S, $a - a + \sqrt{2}$ must be in an irrational number
- $a - a + \sqrt{2} = \sqrt{2}$ which is indeed an irrational number.
- Therefore, is S reflexive

2. **Symmetry**

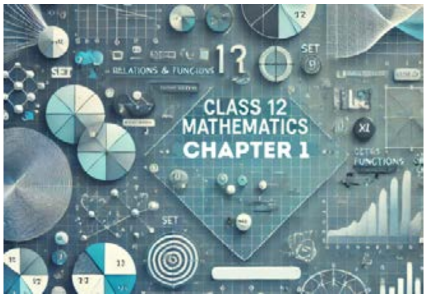
- A relation is S symmetric if for all S $a, b \in R$, if $(a, b) \in S$, then . $(b, a) \in S$
- Suppose $(a, b) \in S$. This means $a - b + \sqrt{2}$ is an irrational number.
- We need to check if $b - a + \sqrt{2}$ is also an irrational number.
- Since the irrationality of a number is not affected by changing the signs of rational numbers, $b - a + \sqrt{2}$ is also irrational.
- Therefore, S is symmetric.

3. **Transitivity:**

- A relation is S transitive if for all S $a, b, c \in R$, if $(a, b) \in S$ and $(b, c) \in S$,
- then $(a, c) \in S$.
- Suppose $(a, b) \in S$ and $(b, c) \in S$. This means $a - b + \sqrt{2}$ is irrational and
- $b - c + \sqrt{2}$ is irrational.
- However, the sum of two irrational numbers can be rational or irrational. Hence, $a - c + 2\sqrt{2}$ is not guaranteed to be irrational.
- Therefore, is S not necessarily transitive.

Conclusion:

The relation S is reflexive and symmetric but not necessarily transitive.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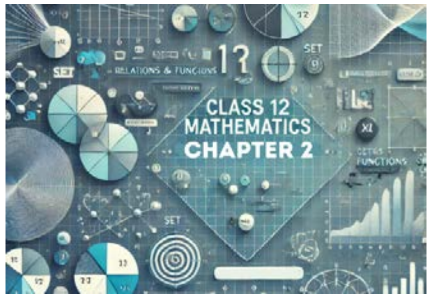
CLASS 12
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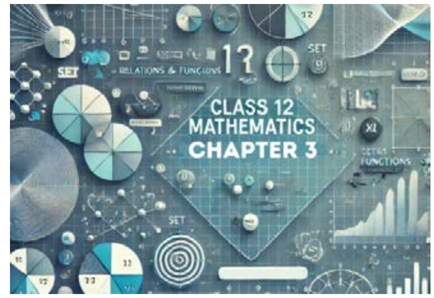
CHAPTER 2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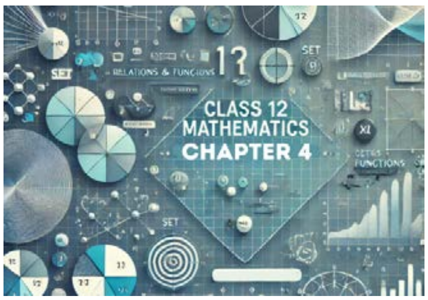
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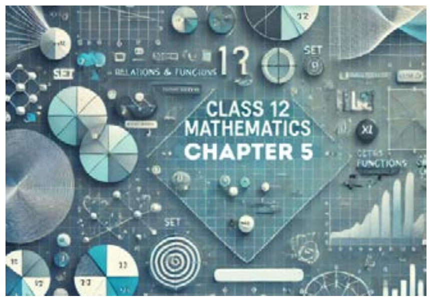
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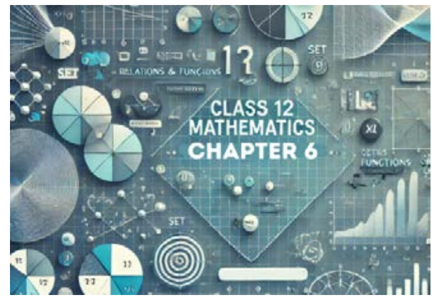
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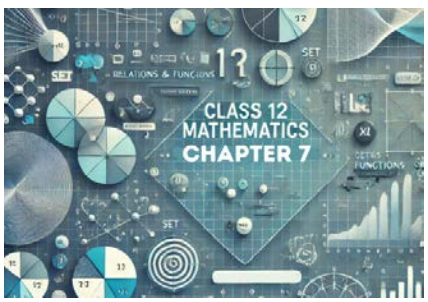
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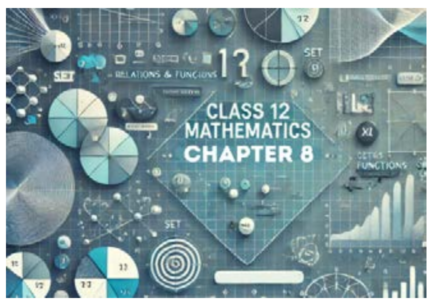
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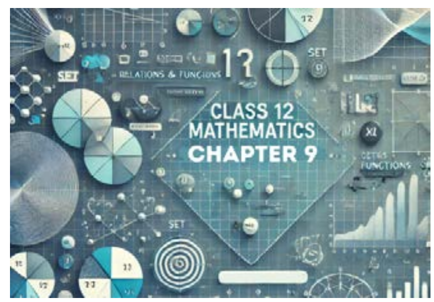
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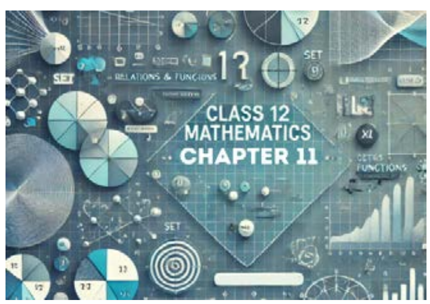
CHAPTER 10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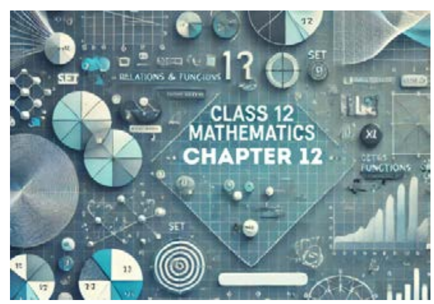
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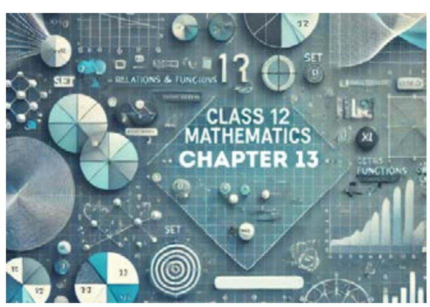
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